

operations, namely, $S_0 + S_1 + \dots + S_{N-1}$, obeys these conditions. This sum is called the complete group, and all other groups are its sub-groups.

The first six sections of this paper are devoted to the detailed establishment of this purely algebraic view of the subject. At times the modification in treatment from that adopted in the standard treatises on the subject, such as Burnside's 'Theory of Groups of Finite Orders,' is slight. Where the modification would be of no sufficient interest it has been simply omitted, and the theorems when wanted have been assumed as part of the general knowledge of the subject. Only so much reasoning has been given as will establish the principles of the Algebra of Groups of Finite Order, viewed as an algebra independent of any interpretation, however vague.

The more special object of this paper follows directly from the changed point of view from which the Theory of Groups is here regarded. The idea of the group is no longer so absorbing; the set takes its place as the fundamental general entity which has to be investigated. A group is a special type of set. Accordingly in this paper some of the general properties of sets are investigated. A set of operations has numerous groups associated with it, and these groups have many relations with each other which this paper cannot pretend to have exhausted. The fundamental idea of this part of the paper (*cf.* § 7) is the formation from a set H of an unending series of other sets, here called the successive powers of H , and in the notation of the algebra written $H^2, H^3, \dots$. This series is called the power sequence of H . Any group which contains H also contains its power sequence. The power sequence is proved to have a periodic property (*cf.* § 9) which introduces a curious analogy to recurring decimals. This periodicity is the foundation of the rest of the paper. It governs the relations to each other of the various allied groups and sets. The periodicity is expressed by an equation of the form

$$H^{n+sm+q} = H^{n+q},$$

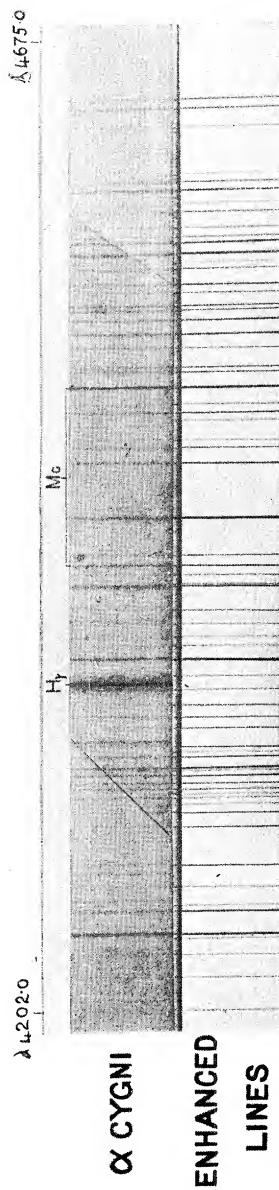
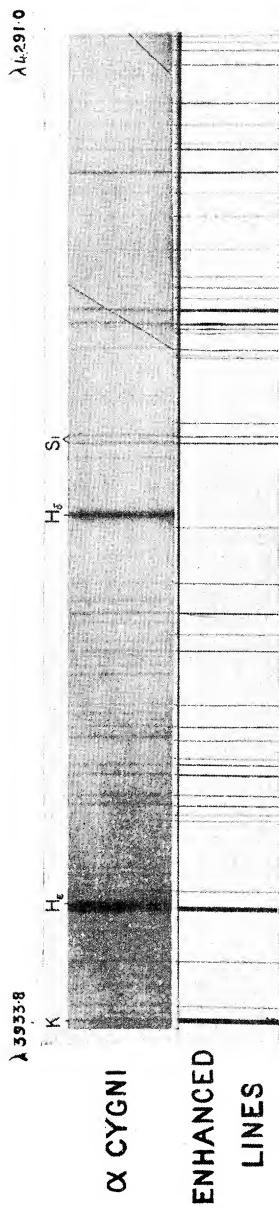
where m is called the period of H , and n the characteristic, and s and q are any integers including zero. The number of theorems relating to m is very large.

"Note on the Enhanced Lines in the Spectrum of α Cygni." By
SIR NORMAN LOCKYER, K.C.B., F.R.S. Received January 20
—Read February 2, 1899.

(PLATE 6.)

When engaged in the classification of stars, according to their photographic spectra, in 1893* I came across two sets of lines of

* 'Phil. Trans.,' A, vol. 184, p. 675.



unknown origin, one in the hottest stars, the other in stars of intermediate temperature.

After the discovery of a terrestrial source of helium by Professor Ramsay, I showed in a series of seven notes communicated to the Royal Society,* May—September, 1895, that the cleveite gases, which I obtained by the process of distillation, accounted to a very great extent for the first set.

In 1897 in a series of three communications to the Royal Society,† I pointed out that some of the other set of unknown lines in the stars of intermediate temperature, taking α Cygni as an example, were due to the enhanced spark lines of iron and other metals, the arc lines being almost entirely absent.

During the last year, this research has been continued; and latterly, by the kindness of Mr. Hugh Spottiswoode, the photographs of the enhanced lines have been obtained by the use of the large induction coil, formerly belonging to Dr. Spottiswoode, P.R.S. I am anxious to express here my deep obligation to Mr. Hugh Spottiswoode for the loan of such a magnificent addition to our instrumental aids.

The spark obtained by means of the Spottiswoode coil, is so luminous that higher dispersions than those formerly employed can be effectively used, and in consequence of this, the detection of the enhanced lines becomes more easy; their number therefore has been considerably increased.

The observations have already been mapped for the following substances :—Fe, Mg, Ca, Si, Sr, Va, Ti, Ni, Mn, Cr, Co, Cu.

In the accompanying photograph, a comparison is shown between the lines of α Cygni and the enhanced lines of the above substances thrown together. The extraordinary number of coincidences is seen at a glance. The facts are as follows :—

The number of lines measured in the spectrum of α Cygni at Kensington between $\lambda 3798\cdot 1$ and $\lambda 4861\cdot 6$ is	307
Of these the number which approximately coincides with the enhanced metallic lines so far observed is.....	120
The number of lines (excluding the hydrogen series) in α Cygni of intensity over 4 (the maximum being represented by 10) is	40
Of this number, the coincidences with enhanced metallic lines with the dispersion employed amount to	38

I shall deal in a subsequent communication, when the enquiry has reached a further stage, with the details for each element.

* 1st Note, 'Roy. Soc. Proc.,' vol. 58, p. 67; 2nd, *ibid.*, vol. 58, p. 113; 3rd, *ibid.*, vol. 58, p. 116; 4th, *ibid.*, vol. 58, p. 192; 5th, *ibid.*, vol. 58, p. 193; 6th, *ibid.*, vol. 59, p. 4; 7th, *ibid.*, vol. 59, p. 342.

† 'Roy. Soc. Proc.,' vol. 60, p. 475; *ibid.*, vol. 61, p. 148; *ibid.*, vol. 61, p. 441.

The lines of the stars of intermediate temperature, like α Cygni, have long been recognised by the Harvard observers as well as by myself as presenting great difficulties.

In 1893 I wrote as follows* :—"With the exception of the K line, the lines of hydrogen and the high temperature line of Mg at $\lambda 4481$, all the lines may be said to be at present of unknown origin. Some of the lines fall near lines of iron, but the absence of the strongest lines indicates that the close coincidences are probably accidental."

In the Harvard 'Spectra of Bright Stars' 1897, p. 5, the following words occur, relating to the same stars :—

"This system of lines should perhaps be regarded as forming a separate class, as in the case of the Orion lines, and should not be described as 'metallic,' as has just been done in the absence of any more distinctive name."

From the fact that these unknown lines have now been traced to a "proto-metallic" origin, as effectively as the unknown lines of the hottest stars have been traced to helium and asterium, we may expect that the consequences of this determination in relation to stellar classification and other connected matters, will be very far reaching. At present I am using this new spectrum consisting of enhanced lines as an explorer, in relation to some further details of stellar classification having special reference to stars of Groups III and IV in which bright as well as dark lines occur.

"On the Effects of Strain on the Thermo-Electric Qualities of Metals." By MAGNUS MACLEAN, M.A., D.Sc. Communicated by Lord KELVIN, F.R.S. Received January 23,—Read February 2, 1899.

1. Seebeck† discovered the great effect that hardness, or softness, or crystalline structure, has on the thermo-electric properties of metals. Magnus made a number of experiments by winding a hard drawn wire on a reel. Parts of this wire were softened and annealed. When heat was applied to the parts of the wire which were between unannealed and annealed, a thermo-electric current was obtained. In this way Magnus found that the current passed from soft to hard through the hot junction for silver, steel, cadmium, copper, gold, and platinum; and that it passed from hard to soft through the hot junction for German silver, zinc, tin, and iron.

2. Lord Kelvin describes in vol. 2 of his 'Mathematical and Physical Papers' a number of qualitative experiments to determine the direction

* 'Phil. Trans.,' A, vol. 184, p. 694.

† 'Pogg. Ann.,' 1826.

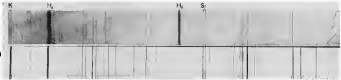
15000

16000

α CYGNI

ENHANCED

LINES



$\lambda_{\text{rest}} 2026$
1

$\lambda_{\text{rest}} 4875$
1

α CYGNI

H _{γ}

H _{β}

ENHANCED

LINES

